

Discrete Mathematics Final Review Spring 2015

Unit 1 Fair Division

1. Suppose that a labor union and management are trying to resolve a dispute that involves four issues: the base salary of the workers, the annual salary increase that workers can expect, the benefits package the workers will receive, and the amount of vacation time to which each worker will be entitled. Suppose they use adjusted winner to resolve this dispute with the following point assignments:

Issue	Labor	Management
Base Salary	50	5
Salary increases	30	50
Benefits	15	30
Vacation time	5	15

Using adjusted winner, what does Labor end up with?

2. Mark and Fred have inherited a number of items from their parents' estate, with no indication of who gets what. They rank the items from most preferred to least as follows:

Mark	Fred
Boat	Car
Tractor	Truck
Car	Motorcycle
Truck	Boat
Motorcycle	Tools
Tools	Tractor

Assume that Mark and Fred use the bottom-up strategy and that Mark gets to choose first. Determine Mark's first choice.

Use for questions 3 – 4

Three heirs, A, B, C, inherit a house in the city, a small farm, and a valuable sculpture, and who submit sealed bids (in dollars) on these objects as follows:

	A	B	C
House	165,000	149,999	168,000
Farm	135,000	135,001	127,999
Sculpture	120,000	80,000	110,000

3. Based on Knaster Inheritance, who would receive the house?

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4. To receive the sculpture, how much would person A need to place in the kitty?

5.. What happens to the money left in the kitty after those people who did not receive the object take out their fair share?

6. In a fair division procedure, participants may receive different amounts. True or false?

7. Tom and Sandy must make a fair division of the three objects left by their Great Aunt Sally. They have assigned points to the objects as shown below. Using the adjusted winner procedure, who is initially awarded the painting?

Object	Tom's Points	Sandy's Points
Painting	40	30
Jewelry	10	50
Car	50	20

8. In the adjusted winner procedure, what happens if there is a tie in the bidding?

9. Envy-free means equal. True or false?

Unit 2 Matrices

10. Suppose Nancy has expanded her jewelry business and now has shops in the Westmarket and Eastmarket plazas as well as in the Oldmarket. Her sales of cultured pearl sets for July are shown in the table below.

	Old	West	East
Earrings	10	8	12
Pins	6	5	4
Necklaces	3	2	2
Bracelets	4	3	2

Earrings sell for \$40 a pair, pins for \$35 each, necklaces for \$80, and bracelets for \$45. Use matrix multiplication to find Nancy's total sales at each location.

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Use for questions 11-14

The characteristics of the female population of a herd of unicorns are shown in the following table.

Age Groups (years)

	0-4	4-8	8-12	12-16	16-20	20-24
Birthrate	0	.5	1.1	.9	.3	0
Survival Rate	.5	.7	.9	.8	.7	0

Suppose the initial female population for the unicorns is:

$$P_0 = [22 \ 22 \ 18 \ 20 \ 7 \ 2]$$

11. What is the expected lifespan of the unicorn?

12. Show the population distribution for P_2 .

13. Show the population distribution for P_4 .

14. How long in years is 4 transitions?

15. Leslie Matrices are square. True or False?

16. The regents at a state university recently announced a 7% raise of tuition rates per semester hour. The current rates per semester are shown in the following table.

	Undergraduate	Graduate
Resident	\$75.00	\$99.25
Nonresident	\$204.00	\$242.25

Find a new matrix that represents the tuition rates per semester hour after the 7% raise goes into effect.

17. If a 3×2 matrix is added to a 2×3 matrix, what would the new dimensions be?

18. If a 3×2 matrix is multiplied to a 2×3 matrix, the resulting matrix's dimensions would be:

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Unit 3 Voting Theory

19. An election with six candidates, $A, B, C, D, E,$ and F is held under the method of pairwise comparisons. Partial results of the pairwise comparisons are: A wins three pairwise comparisons, B and C both win two, D and E both win two and tie one. Find the winner of the election.

20. A perfect voting method exists. True or false?

21. All voting methods produce the same winner. True or false?

22. Consider the election given by the following preference schedule.

# of voters	7	4	2
1 st choice	A	B	D
2 nd choice	B	D	A
3 rd choice	C	C	C
4 th choice	D	A	B

Find the winner of this election under the Borda count method.

Use the following information for questions 23-24

The student body at Eureka High School is having an election for Homecoming Queen. The candidates are Alicia, Brandy, Cleo, and Dionne. The preference schedule for the elections is as follows:

# of voters	153	102	55	202	108	20	110	160	175	155
1 st	A	A	A	B	B	B	C	C	D	D
2 nd	C	B	D	D	C	C	A	B	A	B
3 rd	B	D	C	A	D	A	D	A	C	C
4 th	D	C	B	C	A	D	B	D	B	A

23. Find the winner of the election under the plurality method.

24. Is there a majority winner?

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25. An election is held by the 27 members of the National Football League Executive Committee to choose the host city for the upcoming Super Bowl. The finalists are Atlanta, Boston, Chicago, and Denver. The preference schedule for the election is given in the following table.

# of voters	10	6	5	4	2
1 st choice	A	B	B	C	D
2 nd choice	C	D	C	A	C
3 rd choice	B	C	A	D	B
4 th choice	D	A	D	B	A

Find the winner of the election by pairwise comparison.

26. The candidates for Mathematics Department Chair are Professors Argand, Brandt, Chavez, Dietz, and Epstein. The preference schedule is in the following table.

# of voters	5	3	5	2	2	3
1 st	A	A	C	D	D	B
2 nd	B	D	E	C	C	E
3 rd	C	B	D	B	B	A
4 th	D	C	A	E	A	C
5 th	E	E	B	A	E	D

Suppose the election is to be decided under the plurality with elimination method. Find the winner of the election.

27. Determine whether any of the listed candidates has a majority.

Four candidates running for congress receive votes as follows:

Alberts: 45,771, Brown: 12,483, Cassimatis: 33,288, D'Amico: 29,127

28. In the indicated voting system, the weights represent, in order, voters A, B, C, and so on. Identify the dictator if there is one, and identify those voters with veto power if there are any.

[15 : 1, 2, 3, 3, 4, 5]

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Unit 4 Apportionment**Use for questions 29-30**

A county is divided into three districts with populations shown below. There are 10 seats on the county council which need to be apportioned.

<u>District</u>	<u>Population</u>
Applewood	4376
Boxwood	6262
Central	5222

29. Find the apportionment using the Hamilton Method.

State	Population	Standard Quota	Lower Quota	Fractional Parts	Surplus	Hamilton Apportionment
Applewood						
Boxwood						
Central						
Total						

30. Find the apportionment using the Jefferson method.

State	Population	Standard Quota	Lower Quota	Critical Divisor	Surplus	Jefferson Apportionment
Applewood						
Boxwood						
Central						
Total						

Use for questions 31-32

A health club instructor has a course load that allows her to teach six two credit hour classes. A pre-registration survey indicates the following interests:

- 32 want to take high-impact aerobics
- 64 want to take low-impact aerobics
- 37 want to take jazzercise
- 17 want to take step exercise

31. Find the apportionment using the Hamilton method.

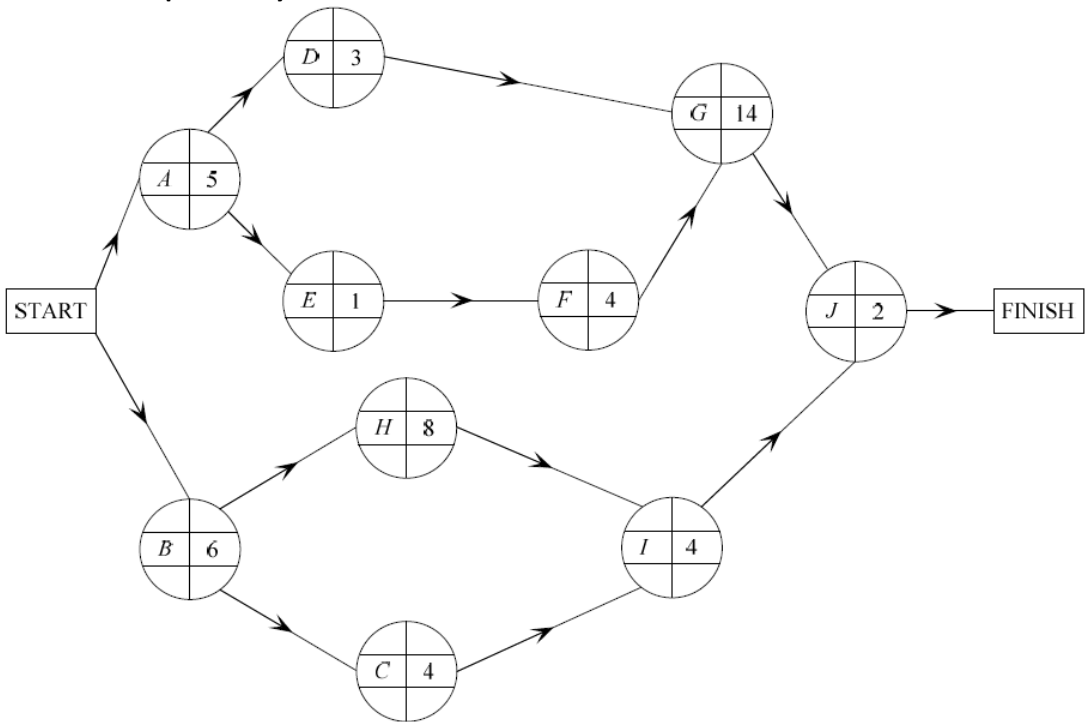
32. For a given set of populations and house sizes, different methods of apportionment always lead to the same apportionments. True or false?

33. What is a violation of upper quota condition?

34. The house has gained seats, but a state loses a seat. Which paradox is this?

35. Which method is the method currently used in the US Congress?

Unit 5 and 6 Graph Theory



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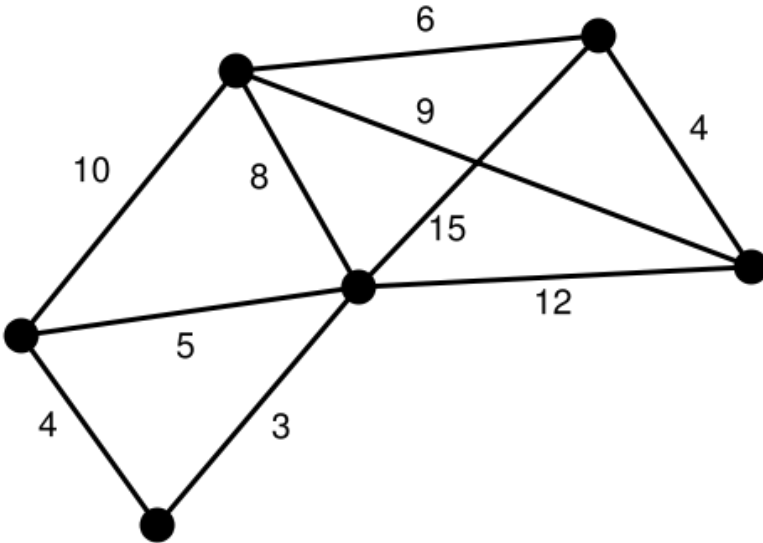
Use for questions 36 – 37.

The digraph below represents time it takes to prepare a taco dinner.

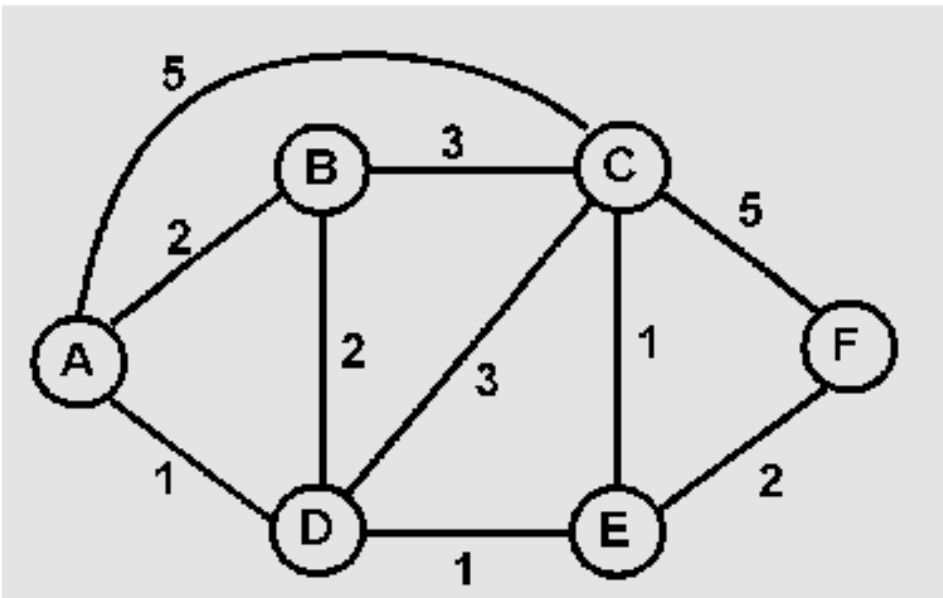
36. How long will it take to make dinner?

37. What is the critical path?

38. What is the total weight of the graph below using Kruskal's algorithm?



Use the following graph for questions 39 - 40



39. Using Dijkstra's Algorithm, find the shortest path to F from A.

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40. Using the shortest path to F from A, how long would it take if the weights represented minutes?

Use for 41 – 43

41. The organizer for a regional Mathematics conference must schedule times for five presentations. A table created from the preliminary sign-up sheets show which of the five presentations the participants wish to attend. **Draw a graph to be used to decide how many different presentation times would be required?**

	A	B	C	D	E
A		X	X	X	X
B	X		X	X	
C	X	X			X
D	X	X			X
E	X		X	X	

42. Using the chart and graph, how many different presentation times would be required?

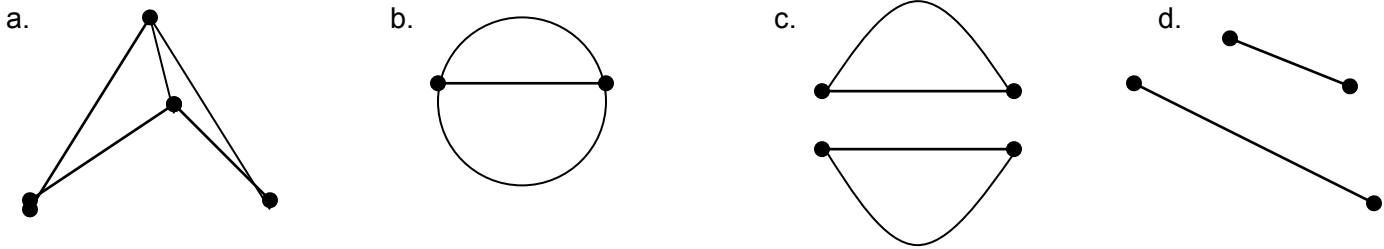
43. Using the chart information, what would the nodes represent on the graph?

44. How many colors are required to color the following map?

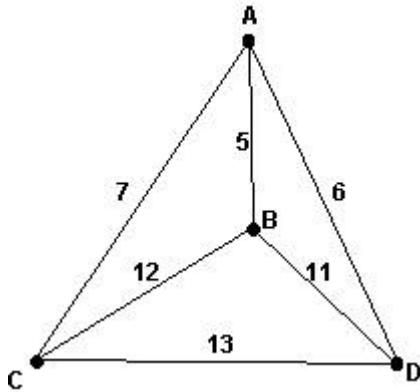


45. Any map can be colored in four or less colors. True or False?

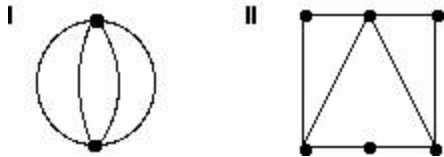
46. Which diagram represents a graph with 4 vertices each with a degree of 2?



47. On the graph below, which routing is produced by using the nearest neighbor algorithm to solve the traveling salesman problem if you have to begin at node B?



48. Which of the graphs below have Euler circuits?



Unit 7 More with Matrices & Binomial Distribution

Markov Chains

Use the following for questions 49 – 51.

The probabilities of the zombie apocalypse at Holly Springs High School are as follows after a two month period of research:

- Human stays a Human = 34%
- Human turns to Zombie = 66%
- Zombie turns to Human = 1%
- Zombie stays a zombie = 99%

49. Create a transition Matrix

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50. There are 3,000 students/teachers at Holly Springs High School. If in a population of 3,000 people, 2000 are human and 1000 are zombies. How many people will be human and how many will be zombies at the end of 2 months?

51. There are 3,000 students/teachers at Holly Springs High School. If in a population of 2,247 people, 2000 are human and 1000 are zombies. How many people will be human and how many will be zombies at the end of the school year (10 months)?

52. Katniss Everdeen has a 90% chance of hitting the bulls eye of a target. What is the probability that she will hit at least 18 out of 20 bulls eyes?

Unit 8 Modeling, Sequences, & Series

53. Assume that the equation $A = -0.125x^2 + 2x + 1.125$ models attendance of a recently released movie. During what week will the movie attain its highest attendance?

54. According to the Bureau of Labor Statistics, the demand for desktop publishing specialists will increase from 26,000 in 1998 to 44,000 in 2008. Assume that this projection is correct. Model this data with a linear equation and estimate the demand for desktop publishing specialists in 2003. What is the demand in 2003?

55. You run a restaurant that sells fried chicken. You can purchase raw chicken parts from the processor for a current wholesale price of \$0.85 per pound. The price increases at the rate of 12 % each year. What will you be paying in 5 years?

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56. A biologist took a count of the number of migrating waterfowl at a particular lake, and recounted the lake's population of waterfowl on each of the next six weeks.

Week	0	1	2	3	4	5	6
Population	585	582	629	726	873	1,070	1,317

a. Find a quadratic function that models the data as a function of x , the number of weeks.

b. Use the model to estimate the number of waterfowl at the lake on week 8.

57. An initial population of 280 quail increases at an annual rate of 17%. Write an exponential function to model the quail population.

58. What is the 10th term of the following sequence 22, 26, 30, 34, 38... ?

59. If the following sequence is geometric, what is the missing term? 2, _____, 8...

Use $\sum_{n=4}^{10} 0.8n - 0.4$ for questions 60 and 61.

60. How many terms are in the sequence?

61. What is the last term of the sequence?

62. Arithmetic series have a common divisor. True or False?

63. Geometric series have a common difference. True or False?

64. Sequences can be both geometric and arithmetic. True or False?

65. If the following sequence is arithmetic, find the missing term, 10, _____, 17