Name:_____

Discrete Mathematics Final Review Spring 2015

Unit 1 Fair Division

1. Suppose that a labor union and management are trying to resolve a dispute that involves four issues: the base salary of the workers, the annual salary increase that workers can expect, the benefits package the workers will receive, and the amount of vacation time to which each worker will be entitled. Suppose they use adjusted winner to resolve this dispute with the following point assignments:

Issue	Labor	Management
Base Salary	50	5
Salary increases	30	50
Benefits	15	30
Vacation time	5	15

Using adjusted winner, what does Labor end up with?

L		M		
Base 50	SI	50	LPR 1.6	← SPLIT
	B	30	2	
50	VT	15	3	
		95		

$50x + 30 + 15 = 50 + 30(1-x)$

$50x + 45 = 80 - 30x$

$80x = 35$

$x = 43.75\%$

LABOR GETS = BASE SALARY & 56.25% OF SALARY INCREASES.

2. Mark and Fred have inherited a number of items from their parents' estate, with no indication of who gets what. They rank the items from most preferred to least as follows:

Mark	Fred
Boat	Car
Tractor	Truck
~~Car~~	~~Motorcycle~~
~~Truck~~	~~Boat~~
Motorcycle	Tools
~~Tools~~	~~Tractor~~

Assume that Mark and Fred use the bottom-up strategy and that Mark gets to choose first. Determine Mark's ~~first choice~~. PICKS

MARK: BOAT, TRACTOR, MOTORCYCLE.

Use for questions 3 – 4

Three heirs, A, B, C, inherit a house in the city, a small farm, and a valuable sculpture, and who submit sealed bids (in dollars) on these objects as follows:

	A	B	C
House	165,000	149,999	168,000
Farm	135,000	135,001	127,999
Sculpture	120,000	80,000	110,000

3. Based on Knaster Inheritance, who would receive the house?

HEIR C BECAUSE THEY BID THE MOST.

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4. To receive the sculpture, how much would person A need to place in the kitty?

$120,000 \cdot \frac{2}{3} = \$80,000$

5.. What happens to the money left in the kitty after those people who did not receive the object take out their fair share?

SPLIT THE SURPLUS BY 3 PEOPLE AND EACH RECEIVE THAT AMOUNT.

6. In a fair division procedure, participants may receive different amounts. True or false?

TRUE!

7. Tom and Sandy must make a fair division of the three objects left by their Great Aunt Sally. They have assigned points to the objects as shown below. Using the adjusted winner procedure, who is initially awarded the painting?

Object	Tom's Points	Sandy's Points
Painting	40	30
Jewelry	10	50
Car	50	20

TOM IS INITIALLY AWARDED THE PAINTING BECAUSE HE BID MORE!

8. In the adjusted winner procedure, what happens if there is a tie in the bidding?

THE ITEM GOES TO THE PERSON WHO HAS THE LEAST AMOUNT OF POINTS.

9. Envy-free means equal. True or false?

FALSE.

Unit 2 Matrices

10. Suppose Nancy has expanded her jewelry business and now has shops in the Westmarket and Eastmarket plazas as well as in the Oldmarket. Her sales of cultured pearl sets for July are shown in the table below.

	Old	West	East
Earrings	10 · 40	8 · 40	12 · 40
Pins	6 · 35	5 · 35	4 · 35
Necklaces	3 · 80	2 · 80	2 · 80
Bracelets	4 · 45	3 · 45	2 · 45

Earrings sell for $40 a pair, pins for $35 each, necklaces for $80, and bracelets for $45. Use matrix multiplication to find Nancy's total sales at each location.

OLD WEST EAST

$\begin{bmatrix} \$1030 & \$790 & \$870 \end{bmatrix}$

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Use for questions 11-14

The characteristics of the female population of a herd of unicorns are shown in the following table.

Age Groups (years)

	0-4	4-8	8-12	12-16	16-20	20-24
Birthrate	0	.5	1.1	.9	.3	0
Survival Rate	.5	.7	.9	.8	.7	0

Suppose the initial female population for the unicorns is:

$P_0 = [22 \quad 22 \quad 18 \quad 20 \quad 7 \quad 2]$

A

11. What is the expected lifespan of the unicorn?

24 YEARS

12. Show the population distribution for P_2.

$A \cdot B^2 =$ [41.82 25.45 7.7 13.86 12.96 11.2]

~~[101.8 22 30.8 32.4 32 9.8]~~

13. Show the population distribution for P_4.

$A \cdot B^4 =$ [39.61 18.77 14.63 ~~7.76~~ 16.03 5.544 7.76]

14. How long in years is 4 transitions?

4 . 4 = 16 years

15. Leslie Matrices are square. True or False?

TRUE

16. The regents at a state university recently announced a 7% raise of tuition rates per semester hour. The current rates per semester are shown in the following table.

	Undergraduate	Graduate
Resident	$75.00	$99.25
Nonresident	$204.00	$242.25

Find a new matrix that represents the tuition rates per semester hour after the 7% raise goes into effect.

U G

R [$80.25 $106.19]

NR [$218.28 $259.20]

17. If a 3X2 matrix is added to a 2X3 matrix, what would the new dimensions be?

CANT ADD THOSE TWO MATRICES BECAUSE THEY DO NOT HAVE SAME DIMENSIONS.

18. If a 3X2 matrix is multiplied to a 2X3 matrix, the resulting matrix's dimensions would be:

3x2 · 2x3 = [3x3]

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Unit 3 Voting Theory

19. An election with six candidates, *A*, *B*, *C*, *D*, *E*, and *F* is held under the method of pairwise comparisons. Partial results of the pairwise comparisons are: *A* wins three pairwise comparisons, *B* and *C* both win two, *D* and *E* both win two and tie one. Find the winner of the election.

A WINS THE ELECTION

20. A perfect voting method exists. True or false?

FALSE

21. All voting methods produce the same winner. True or false?

FALSE

22. Consider the election given by the following preference schedule.

# of voters	**7**	**4**	**2**
1st choice	*A*	*B*	*D*
2nd choice	*B*	*D*	*A*
3rd choice	*C*	*C*	*C*
4th choice	*D*	*A*	*B*

Find the winner of this election under the Borda count method.

A = 38 PTS

B = 39 PTS

C = 39 PTS

D = 27 PTS

TIE BTWN B & C

Use the following information for questions 23-24

The student body at Eureka High School is having an election for Homecoming Queen. The candidates are Alicia, Brandy, Cleo, and Dionne. The preference schedule for the elections is as follows:

# of voters	**153**	**102**	**55**	**202**	**108**	**20**	**110**	**160**	**175**	**155**
1st	*A*	*A*	*A*	*B*	*B*	*B*	*C*	*C*	*D*	*D*
2nd	*C*	*B*	*D*	*D*	*C*	*C*	*A*	*B*	*A*	*B*
3rd	*B*	*D*	*C*	*A*	*D*	*A*	*D*	*A*	*C*	*C*
4th	*D*	*C*	*B*	*C*	*A*	*D*	*B*	*D*	*B*	*A*

23. Find the winner of the election under the plurality method.

A = 310 VOTES B = 330 VOTES C = 270 VOTES D = 330 VOTES

TIE BTWN B & D

24. Is there a majority winner?

NO

Name:_____

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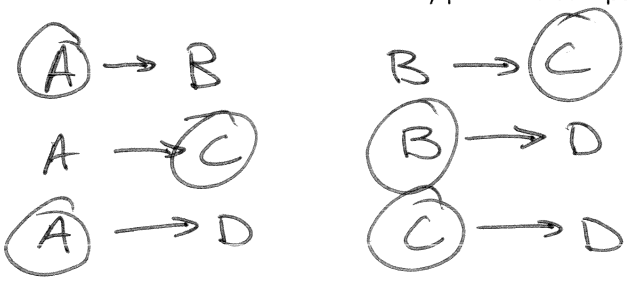
25. An election is held by the 27 members of the National Football League Executive Committee to choose the host city for the upcoming Super Bowl. The finalists are Atlanta, Boston, Chicago, and Denver. The preference schedule for the election is given in the following table.

26 VOTES

# of voters	10	6	5	4	2
1st choice	A	B	B	C	D
2nd choice	C	D	C	A	C
3rd choice	B	C	A	D	B
4th choice	D	A	D	B	A

Find the winner of the election by pairwise comparison.



C WINS ELECTION w/ 3 POINTS.

26. The candidates for Mathematics Department Chair are Professors Argand, Brandt, Chavez, Dietz, and Epstein. The preference schedule is in the following table.

# of voters	5	3	5	2	2	3
1st	A	A	C	D	D	~~B~~
2nd	~~B~~	D	~~E~~	C	C	~~E~~
3rd	C	~~B~~	D	~~B~~	~~B~~	A
4th	D	C	A	~~E~~	A	C
5th	~~E~~	~~E~~	~~B~~	A	~~E~~	D

Suppose the election is to be decided under the plurality with elimination method. Find the winner of the election.

A = 8 ~~E = 0~~ ELIMINATE

~~B = 3~~ ← ELIMINATE

C = 5

D = 4

A = 11 ← WINNER

C = 5

D = 4

27. Determine whether any of the listed candidates has a majority.

Four candidates running for congress receive votes as follows:

Alberts: 45,771, Brown: 12,483, Cassimatis: 33,288, D'Amico: 29,127

120,669 TOTAL VOTES

MAJORITY IS 60,335

NO MAJORITY.

28. In the indicated voting system, the weights represent, in order, voters A, B, C, and so on. Identify the dictator if there is one, and identify those voters with veto power if there are any.

[15 : 1, 2, 3, 3, 4, 5]

MAJORITY IS 8 VOTES

THEREFORE NO DICTATOR OR VETO POWER

Name:_____

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Unit 4 Apportionment

Use for questions 29-30

A county is divided into three districts with populations shown below. There are 10 seats on the county council which need to be apportioned.

District	Population
Applewood	4376
Boxwood	6262
Central	5222

29. Find the apportionment using the Hamilton Method.

SD = 1586

State	Population	Standard Quota	Lower Quota	Fractional Parts	Surplus	Hamilton Apportionment
Applewood	4376	2.75	2	.75	1	3
Boxwood	6262	3.94	3	.94	1	4
Central	5222	3.29	3	.29		3
Total	15860	—	8			10

30. Find the apportionment using the Jefferson method.

SD = 1586

State	Population	Standard Quota	Lower Quota	Critical Divisor	Surplus	Jefferson Apportionment
Applewood	4376	2.75	2	1458.66	1	3
Boxwood	6262	3.94	3	1252.4 ~~1566.5~~	1	4
Central	5222	3.29	3	1305.5		3
Total	15860	—	8			10

Use for questions 31-32

A health club instructor has a course load that allows her to teach six two credit hour classes. A pre-registration survey indicates the following interests:

32 want to take high-impact aerobics
64 want to take low-impact aerobics
37 want to take jazzercise
17 want to take step exercise

Name:_____

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31. Find the apportionment using the Hamilton method.

	POP	SQ	LQ	FP		SD = 25
HI	32	1.28	1	.28		1
LI	64	2.56	2	.56	1	3
J	37	1.48	1	.48		1
S	17	.68	0	.68	1	1
	150					

32. For a given set of populations and house sizes, different methods of apportionment always lead to the same apportionments. True or false?

FALSE.

33. What is a violation of upper quota condition?

WHEN A STATE RECEIVES MORE SEATS THAN ITS UPPER QUOTA STATES.

34. The house has gained seats, but a state loses a seat. Which paradox is this?

ALABAMA PARADOX

35. Which method is the method currently used in the US Congress?

HILL HUNTINGTON

Unit 5 and 6 Graph Theory

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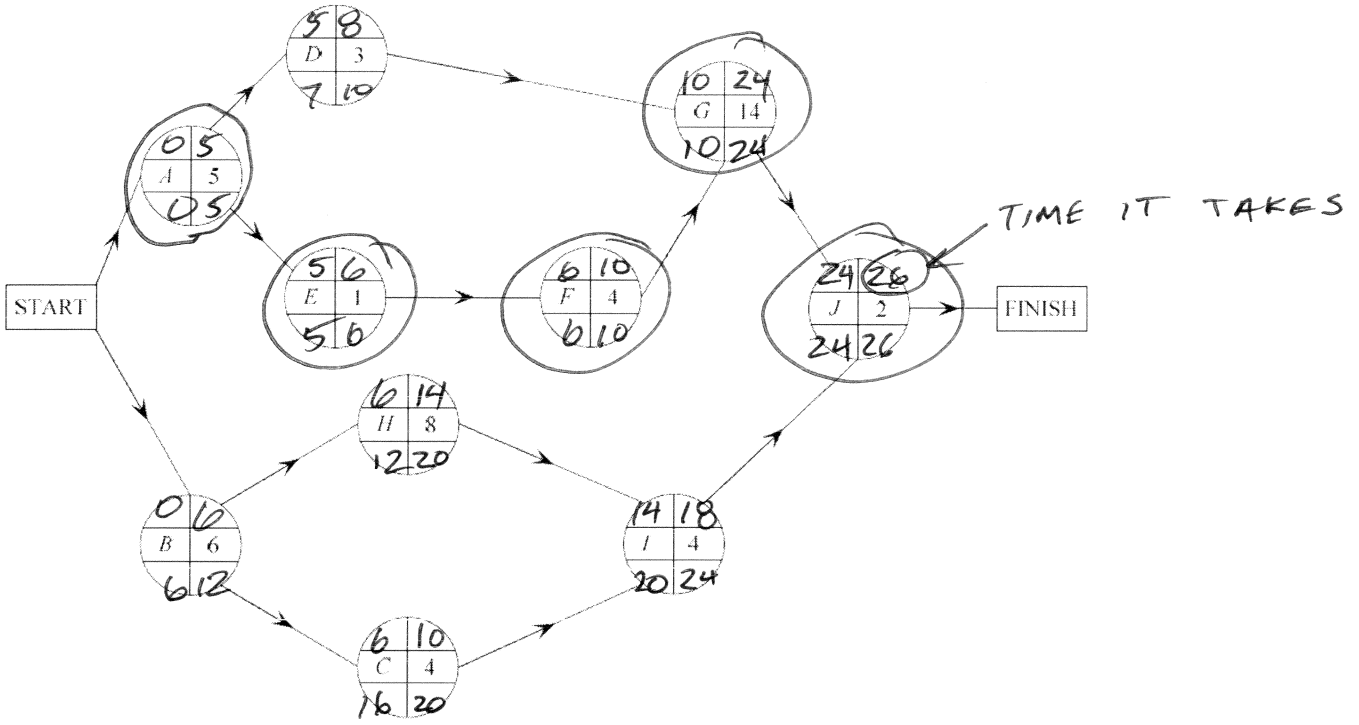
Use for questions 36 – 37.



The digraph below represents time it takes to prepare a taco dinner.

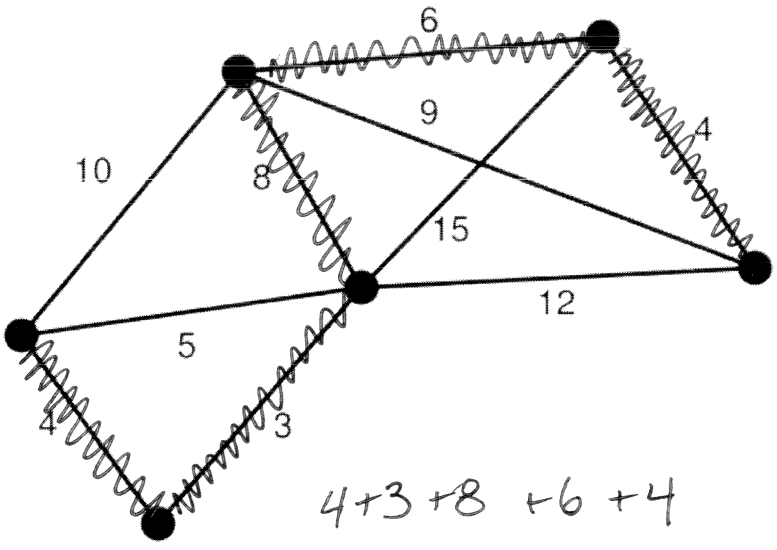
36. How long will it take to make dinner?

26 MINUTES.

37. What is the critical path?

A - E - F - G - J

38. What is the total weight of the graph below using Kruskal's algorithm?



4+3+8+6+4

25

Use the following graph for questions 39 - 40

Name:_____

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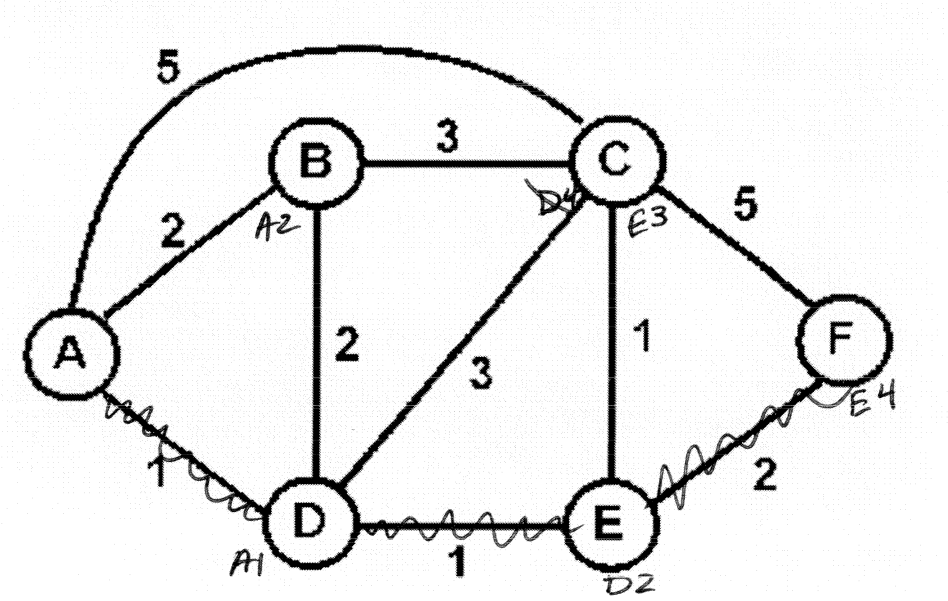


39. Using Dijkstra's Algorithm, find the shortest path to F from A.

ADEF IN 4

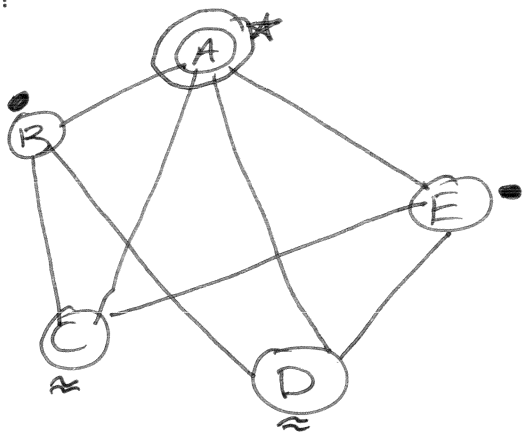
40. Using the shortest path to F from A, how long would it take if the weights represented minutes?

4 MINUTES

Use for 41 – 43

41. The organizer for a regional Mathematics conference must schedule times for five presentations. A table created from the preliminary sign-up sheets show which of the five presentations the participants wish to attend. **Draw a graph to be used to decide how many different presentation times would be required?**

	A	B	C	D	E
A		X	X	X	X
B	X		X	X	
C	X	X			X
D	X	X			X
E	X		X	X	



42. Using the chart and graph, how many different presentation times would be required?

3 PRESENTATION.

43. Using the chart information, what would the nodes represent on the graph?

DIFFERENT TYPES OF PRESENTATIONS.

44. How many colors are required to color the following map?

Name:_____

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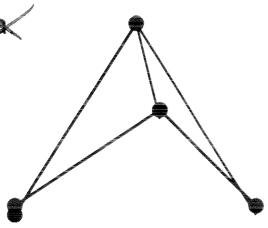
3 COLORS

45. Any map can be colored in four or less colors. True or False?

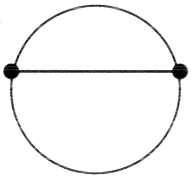
TRUE.

46. Which diagram represents a graph with 4 vertices each with a degree of 2?

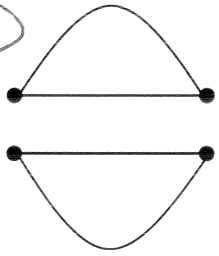
a.



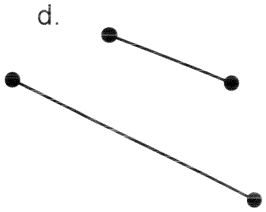
b.



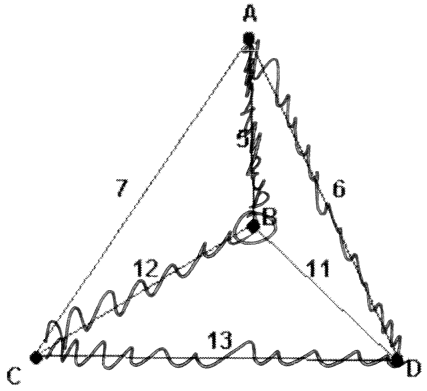
c.



d.



47. On the graph below, which routing is produced by using the nearest neighbor algorithm to solve the traveling salesman problem if you have to begin at node B?



B - A - D - C - B

48. Which of the graphs below have Euler circuits?

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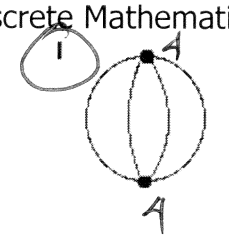

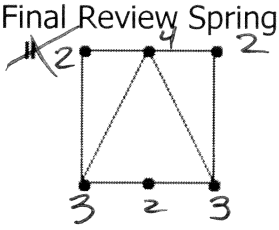

Unit 7 More with Matrices & Binomial Distrubution

Markov Chains

Use the following for questions 49 — 51.

The probabilities of the zombie apocalypse at Holly Springs High School are as follows after a two **month period of research:**

Human stays a Human = 34%
Human turns to Zombie = 66%
Zombie turns to Human = 1%
Zombie stays a zombie = 99%

49. Create a transition Matrix

$$\begin{matrix} & H & Z \\ H & .34 & .66 \\ Z & .01 & .99 \end{matrix}$$

50. There are 3,000 students/teachers at Holly Springs High School. If in a population of 3,000 people, 2000 are human and 1000 are zombies. How many people will be human and how many will be zombies at the end of 2 months?

$$[2000 \quad 1000]\begin{bmatrix} .34 & .66 \\ .01 & .99 \end{bmatrix} = \begin{matrix} H & Z \\ [690 & 2310] \end{matrix}$$

51. There are 3,000 students/teachers at Holly Springs High School. If in a population of 2,247 people, 2000 are human and 1000 are zombies. How many people will be human and how many will be zombies at the end of the school year (10 months)?

$$[A][B]^5 = \begin{matrix} H & Z \\ [52 & 2948] \end{matrix}$$

52. Katniss Everdeen has a 90% chance of hitting the bulls eye of a target. What is the probability that she will hit at least 18 out of 20 bulls eyes?

BINOMPDF(20, .9, 18) = 28.51%
(20, .9, 19) = 27.01%
(20, .9, 20) = 12.
TOTAL = 67.67%

Unit 8 Modeling, Sequences, & Series

53. Assume that the equation $A = -0.125x^2 + 2x + 1.125$ models attendance of a recently released movie. During what week will the movie attain its highest attendance?

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WEEK 8

54. According to the Bureau of Labor Statistics, the demand for desktop publishing specialists will increase from 26,000 in 1998 to 44,000 in 2008. Assume that this projection is correct. Model this data with a linear equation and estimate the demand for desktop publishing specialists in 2003. What is the demand in 2003?

$y = 1800x - 3570400$

↑ 2003

$y = 35000$

55. You run a restaurant that sells fried chicken. You can purchase raw chicken parts from the processor for a current wholesale price of $0.85 per pound. The price increases at the rate of 12 % each year. What will you be paying in 5 years?

year	0	1	2	3	4	5
	.85	.952	1.06	1.19	1.33	1.49

1.49 per pound.

56. A biologist took a count of the number of migrating waterfowl at a particular lake, and recounted the lake's population of waterfowl on each of the next six weeks.

Week	0	1	2	3	4	5	6
Population	585	582	629	726	873	1,070	1,317

a. Find a quadratic function that models the data as a function of x, the number of weeks.

$y = 25x^2 - 28x + 585$

b. Use the model to estimate the number of waterfowl at the lake on week 8.

1961 waterfowl in week 8.

57. An initial population of 280 quail increases at an annual rate of 17%. Write an exponential function to model the quail population.

$y = 280(1.17)^x$

58. What is the 10^{th} term of the following sequence 22, 26, 30, 34, 38... ?

$a_{10} = 58$

59. If the following sequence is geometric, what is the missing term? 2, ____4____, 8...

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Use $\sum_{n=4}^{10} 0.8n - 0.4$ **for questions 60 and 61.**

60. How many terms are in the sequence?

7 terms

61. What is the last term of the sequence?

$0.8(10) - 0.4$ 7.6

62. Arithmetic series have a common divisor. True or False?

FALSE

63. Geometric series have a common difference. True or False?

FALSE

64. Sequences can be both geometric and arithmetic. True or False?

TRUE

65. If the following sequence is arithmetic, find the missing term, 10, __13.5__, 17